THE FINITENESS OF THE INJECTIVE DIMENSION OF MODULES OVER LOCAL RINGS

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This talk is based on joint work with Yuki Mifune and Kenta Shimizu [1].

Let R be a commutative noetherian local ring. Roberts [3] proved that R is Cohen–Macaulay if there exists a nonzero Cohen–Macaulay R-module of finite projective dimension. Following this, Takahashi [4] showed that R is Gorenstein if the type of R is one and there exists a nonzero Cohen–Macaulay R-module of finite G-dimension. Recently, Rahmani and Taherizadeh [2] generalized this result as follows.

Theorem (Rahmani and Taherizadeh)**.** *A semidualizing R-module C is dualizing if the type of C is one and there exists a nonzero Cohen–Macaulay R-module of finite G_C-dimension.*

In this talk, we present a characterization of the finiteness of the injective dimension of a finitely generated R-module in terms of the existence of a Cohen–Macaulay R-module that satisfies an inequality concerning multiplicity and type, together with vanishing of finitely many Ext modules. Our result recovers that of Rahmani and Taherizadeh and yields further applications.

References

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